

Complete $O(\alpha_s^2)$ Corrections to (2+1) Jet Cross Sections in Deep Inelastic Scattering

T. Brodtkorb^a and E. Mirkes^b

^a *Laboratoire Physique Theorique et Hautes Energies, Université Paris-Sud, 91405 Orsay, France. Address after June 1993, SAP Walldorf, Germany*

^b *Physics Department, University of Wisconsin, Madison WI 53706, USA*

Abstract

Complete next-to leading order QCD predictions for (2+1) jet cross sections and jet rates in deep inelastic scattering (DIS) based on a new parton level Monte Carlo program are presented. All relevant helicity contributions to the total cross section are included. Results on total jet cross sections as well as differential distributions in the basic kinematical variables x , W^2 and Q^2 are shown for HERA energies and for the fixed target experiment E665 at FERMILAB. We study the dependence on the choices of the renormalization scale μ_R and the factorization scale μ_F and show that the NLO results are much less sensitive to the variation of $\mu = \mu_F = \mu_R$ than the LO results. The effect of an additional p_T cut to our jet definition scheme is investigated.

I. INTRODUCTION

The start-up of the HERA electron-proton collider in 1992 marked the beginning of a new era of experiments exploring Deep Inelastic Scattering of electrons and protons [1,2]. One of the topics to be studied at HERA will be the deep inelastic (\equiv high Q^2) production of multi jet events*, where good event statistics are expected allowing for precision tests of QCD. Multijet events in DIS are first observed at the FERMILAB E665 experiment [3].

In this paper, we present complete results for (2+1) jet cross sections ("+" denotes the remnant jet) in DIS based on a new parton level Monte Carlo program **DISJET** [4]. All helicity contributions to the total cross section (i.e. $\sigma_{U+L}[(2+1) - \text{jet}]$ and $\sigma_L[(2+1) - \text{jet}]$; see below) are included. Note, that in contrast to jet production in e^+e^- experiments, it is not sufficient to calculate the contraction with the metrical tensor $-g_{\mu\nu}$ on the hadronic tensor. For our kinematical ranges at HERA (defined below), the ratio of " $-g_{\mu\nu}H^{\mu\nu}$ / complete" is 0.75 (0.85) for the "low (high) Q^2 range". Leading order (LO) and next-to leading order (NLO) matrix elements for the processes $eP \rightarrow e + n \text{ jets} + \text{remnant jet}$ ($n = 1, 2, 3$) are implemented in **DISJET**. Using an invariant jet definition scheme introduced in [5,7] we present results for total NLO (2+1) jet cross sections as well as differential distributions in the basic kinematical variables x, W^2 and Q^2 . The dependence on the renormalization scale μ_R and factorization scale μ_F is investigated. It is shown that the NLO predictions are much less sensitive to the choice of the scales than the LO results alone. Varying $\mu^2 = \mu_R^2 = \mu_F^2$ in the range of $1/4p_T^2$ to $4p_T^2$ induces an uncertainty of roughly $\pm 3\%$ in the NLO (2+1) jet cross section predictions compared to about $\pm 17\%$ for the LO results for the "high Q^2 range" ($Q^2 > 100\text{GeV}^2$) at HERA. We have also studied the dependence on other choices of the scales [$\mu^2 = 0.1Q^2 - 10Q^2; y_{\text{cut}}W^2$] and found that these variations lead to larger uncertainties (see table 1). However, a choice of the scales like $\mu^2 \sim p_T^2$ seems to be the more appropriate choice in the case of (2+1) jet production†. We also explore the dependence of the (2+1) jet cross section on an additional cut on the transverse momentum p_T of the jets (p_T is defined with respect to the γ^* direction). It is shown that most of jets at a center of mass (cm)

*Much higher effective luminosity will be achieved for low Q^2 quasi-real photoproduction of jets. The theoretical interpretation of these events, however, is more difficult, since there is added complication of the direct and the resolved photon contribution to the cross section, which are hard to separate

†An extreme example would be (2+1) jet production at very low Q^2 ($Q^2 \approx 1\text{GeV}^2$), where Q^2 cannot be the relevant scale for high p_T jet production.

energy of $\sqrt{s} = 30$ GeV (E665 experiment) are produced with a transverse momentum less than 3 GeV using our jet definition scheme. Therefore, one should use resummation techniques to obtain a reliable perturbation expansion in this region. This problem is not tackled in this paper. Finally we also present results for different values of the jet resolution parameter y_{cut} . A comparison of jet measurements with our predicted QCD results provides a direct tool of determining α_s or $\Lambda_{\overline{\text{MS}}}$. Hadronization corrections may be minimized by restricting such an analyses to large momentum transfer Q^2 which causes sufficiently large transverse momenta of the participating partons.

The paper is organized as follows: In section II we discuss the general structure of the cross sections used in the calculation. Section III explains the jet definition scheme and in section IV we discuss kinematical ranges and numerical results for (2+1) jet cross sections and rates. Finally, section V contains a short summary.

II. MATRIX ELEMENTS

Consider deep inelastic electron proton scattering

$$e^-(l) + \text{proton}(P) \rightarrow \text{proton remnant}(p_r) + \text{parton } 1(p_1) \dots + \text{parton } n(p_n) \quad (1)$$

Reaction (1) proceeds via the exchange of an intermediate vector boson $V = \gamma^*, Z, W$. In this paper only the exchange by a virtual photon is considered. We denote the γ^* -momentum by q , the absolute square by Q^2 , the center of mass energy by s , the square of the final hadronic mass by W^2 and introduce the scaling variables x and y :

$$\begin{aligned} q &= l - l' \\ Q^2 &\equiv -q^2 = xys > 0 \\ s &= (P + l)^2 \\ W^2 &\equiv P_f^2 = (P + q)^2 \\ x &= \frac{Q^2}{2Pq} \quad (0 < x \leq 1) \\ y &= \frac{Pq}{Pl} \quad (0 < y \leq 1) \end{aligned} \quad (2)$$

At fixed s , only two variables in (2) are independent, since e. g.

$$xW^2 = (1 - x)Q^2, \quad Q^2 = xys.$$

When one of the hadronic final state momenta p_1 is measured, reaction (1) is described in the one photon exchange by five parity conserved hadronic structure functions $H_1 - H_5$ [5]

$$\begin{aligned}
H^{\mu\nu} = & H_1 \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + H_2 \frac{1}{Pq} \hat{P}^\mu \hat{P}^\nu + H_3 \frac{1}{Pq} \hat{p}_1^\mu \hat{p}_1^\nu \\
& + H_4 \frac{1}{Pq} \left(\hat{P}^\mu \hat{p}_1^\nu + \hat{P}^\nu \hat{p}_1^\mu \right) + H_5 \frac{1}{Pq} \left(\hat{P}^\mu \hat{p}_1^\nu - \hat{P}^\nu \hat{p}_1^\mu \right)
\end{aligned} \tag{3}$$

where we have introduced current conserved momenta variables $\hat{p}_i^\mu = p_i^\mu - \frac{(p_i q)}{q^2} q^\mu$. The four $\mu \leftrightarrow \nu$ symmetric structure functions $H_1 - H_4$ contribute to so called T-even observables whereas the $\mu \leftrightarrow \nu$ antisymmetric structure function H_5 gives a contribution to “T-odd” observables [6]. To $O(\alpha_s)$ in QCD one populates only $H_1 - H_4$ as there are no loop contributions to that order. LO contributions to these structure functions have been extensively studied in the literature [5,7–14].

Note that in the totally inclusive case, where no final hadron momenta are measured, one only has contributions to H_1 and H_2 which are then denoted by the more familiar names W_1 and W_2 .

Experimentally one can measure the so called helicity cross sections σ_{U+L} , σ_L , σ_T , σ_I , σ_A through lepton hadron correlation effects. They factorize the following y and ϕ dependence [5]:

$$\begin{aligned}
d\sigma[n - \text{jet}] \sim & \left[(1 + (1 - y)^2) d\sigma_{U+L}[n - \text{jet}] - y^2 d\sigma_L[n - \text{jet}] \right. \\
& + 2(1 - y) \cos 2\phi d\sigma_T[n - \text{jet}] - (2 - y) \sqrt{1 - y} \cos \phi d\sigma_I[n - \text{jet}] \\
& \left. + y \sqrt{1 - y} \sin \phi d\sigma_A[n - \text{jet}] \right]
\end{aligned} \tag{4}$$

In eq. (4) ϕ denotes the azimuthal angle between the parton plane (\vec{p}, \vec{p}_1) and the lepton plane (\vec{l}, \vec{l}') (in the $(\gamma^*$ -initial parton)-cms).

The helicity cross sections σ_X ($X \in \{U + L, L, T, I, A\}$) are linearly related to polarization density matrix elements of the virtual γ^* . One has:

$$\sigma_{U+L} \sim h_{00} + h_{++} + h_{--} \tag{5}$$

$$\sigma_L \sim h_{00} \tag{6}$$

$$\sigma_T \sim h_{+-} + h_{-+} \tag{7}$$

$$\sigma_I \sim h_{+0} + h_{0+} - h_{-0} - h_{0-} \tag{8}$$

$$\sigma_A \sim h_{+0} + h_{0+} + h_{-0} + h_{0-} \tag{9}$$

where $h_{mm'} = \epsilon_\mu^*(m) H^{\mu\nu} \epsilon_\nu(m')$, $(m, m' = +, 0, -)$ and $\epsilon_\mu(\pm)(\epsilon_\mu(0))$ are the transversal (longitudinal) polarization vectors of the γ^* in the $(\gamma^*$ -initial parton)-cms[†]. Therefore σ_{U+L} and σ_L labels the unpolarized and longitudinal polar-

[†]The lepton hadron scattering process may be regarded as the scattering of a polarized off-shell gauge boson on the proton where the polarization of the gauge boson is tuned by the scattered lepton’s momentum direction.

ization, σ_T and σ_I, σ_A mean transverse and transverse-longitudinal interference, respectively. These helicity cross sections are also linearly related to the five covariant structure functions defined in eq. (3) (see Appendix B of [5]).

In this paper we present results for (2+1) jet cross sections where we have integrated over the azimuthal angle ϕ . Therefore only σ_{U+L} and σ_L contribute in eq. (4). These two cross sections can technically be obtained by the following covariant projections on the (partonic) hadrontensor $\hat{H}^{\mu\nu}$, which is calculated in fixed order perturbation theory ($p = \eta P$ denotes the momentum of the incoming parton and $x_p = Q^2/(2pq)$):

$$\sigma_{U+L} = \left(-\frac{1}{2}g_{\mu\nu} + \frac{3x_p}{pq}p_\mu p_\nu \right) \hat{H}^{\mu\nu}[n - \text{jet}] \quad (10)$$

$$\sigma_L = \frac{2x_p}{pq}p_\mu p_\nu \hat{H}^{\mu\nu}[n - \text{jet}] \quad (11)$$

The $O(\alpha_s^2)$ (2+1) jet matrix elements represent a full NLO calculation including virtual and real corrections. The following subprocesses contribute to (2+1) jet cross sections up to $O(\alpha_s^2)$:

$$\begin{aligned} \hat{H}^{\mu\nu}[\text{tree}, O(\alpha_s)] : & \gamma^* + q \rightarrow q + G \\ & \gamma^* + G \rightarrow q + \bar{q} \\ \hat{H}^{\mu\nu}[\text{tree}, O(\alpha_s^2)] : & \gamma^* + q \rightarrow q + G + G \\ & \gamma^* + q \rightarrow q + \bar{q} + q \\ & \gamma^* + G \rightarrow q + \bar{q} + G \\ \hat{H}^{\mu\nu}[\text{virtual}, O(\alpha_s^2)] : & \gamma^* + q \rightarrow q + G \\ & \gamma^* + G \rightarrow q + \bar{q} \end{aligned} \quad (12)$$

and the corresponding anti-quark processes with $q \leftrightarrow \bar{q}$. Matrix elements for the *complete* contributions to (2+1) jet NLO $O(\alpha_s^2)$ corrections are first discussed in [15]. The NLO matrix elements used in the MC program **DISJET** are based on these matrix elements. In addition, we have also included the full NLO scale dependent contributions. Note, that the second projection ($\sim p_\mu p_\nu$) in σ_{U+L} in eq. (10) gives a contribution of the order of 15 – 30% to the (2+1) jet cross sections depending on the kinematical ranges, whereas the contribution from σ_L in eq. (11) is fairly small (less than 1% in our kinematical ranges). This originates from the y dependent coefficients $(1 + (1 - y)^2)$ and $-y^2$ and the fact that y is peaked at small values (see fig. 9 in [7]). The (2+1) jet NLO contributions originating from the projection with $-g_{\mu\nu}$ on the hadron tensor (see eq. (10)) are first presented and discussed in detail in [16]. A complete list of tree level matrix elements with up to four partons in the final state can also be found in [17].

The general structure of the NLO jet cross sections in DIS within the framework of perturbative QCD is given by:

$$d\sigma^{had}[(2+1) - \text{jet}] = \int d\eta f_a(\eta, \mu_F^2) d\hat{\sigma}^a(p = \eta P, \alpha_s^2(\mu_R^2), \mu_R^2, \mu_F^2) \quad (13)$$

where one sums over $a = q, \bar{q}, g$. $f_a(\eta, \mu_F^2)$ is the probability density to find a parton a with fraction η in the proton if the proton is probed at a scale μ_F^2 . $\hat{\sigma}^a$ denotes the partonic cross section from which collinear initial state singularities have been factorized out at a scale μ_F^2 and implicitly included in the scale dependent parton densities $f_a(\eta, \mu_F^2)$.

Let us briefly discuss some technical matters that go into the NLO calculation. The $\mathcal{O}(\alpha_s^2)$ tree graph contributions in eq. (12) are integrated over the unresolved phase space region which are (2+1) jet like as defined in eq. (14). Infrared (IR) as well as collinear (M) divergencies associated with the final state partons are cancelled against corresponding IR/M divergencies of the one-loop contributions. The remaining collinear initial state divergencies are factorized into the bare parton densities introducing a factorization scale dependence μ_F . Finally the ultraviolet (UV) divergencies are removed by \overline{MS} renormalization which introduces a renormalization scale dependence μ_R .

III. JET DEFINITION

In order to calculate the (2+1) jet cross section we have to *define* what we call (2+1) jets by introducing a *resolution criterion*. As has been elaborated in detail in [5,7] energy-angle cuts are not suitable for an asymmetric machine with its strong boosts from the hadronic cms to the laboratory frame. As a jet resolution criterion we use the invariant mass cut criterion defined in [5,7,15,16] such that

$$s_{ij} = (p_i + p_j)^2 \geq M^2 = \max \{ y_{cut} M_c^2, M_0^2 \} \quad (i, j = 1, \dots, n, r; \quad i \neq j) \quad (14)$$

where y_{cut} is the resolution parameter and s_{ij} is the invariant mass of any two final state partons, including the remnant jet with momentum $p_r = (1 - \eta)P$. M_c is a typical mass scale of the process which defines the jet definition scheme. In this paper we choose $M_c^2 = W^2$. This corresponds to the "W-" scheme in [5]. Other jet definition schemes based on a k_T algorithm are proposed in [18]. M_0 is a fixed mass cut which we have introduced in order to clearly separate the perturbative and non-perturbative regime in the case where W^2 is small. M_0 is fixed to 2 GeV [7] in all our results.

IV. NUMERICAL RESULTS

We will now turn to our numerical cross section results and present results for the actual HERA cm energy of 295 GeV as well as for the FERMILAB fixed target experiment E665 with a cm energy of 30 GeV. All results are based on a new Monte Carlo program **DISJET** [4]. The Monte-Carlo routines are using the VEGAS-package [19] for numerical integration. Parton distributions are incorporated from the packages [20,21]. Our standard set of parton distribution functions is MRS set D- [22]. If not stated otherwise, we use the one-loop formula for the strong coupling constant α_s for our LO results

$$\alpha_s^{\overline{MS}}(\mu_R^2) = \frac{12\pi}{(33 - 2n_f) \ln \frac{\mu_R^2}{\Lambda^2}} \quad (15)$$

whereas we employ the two loop formula

$$\alpha_s^{\overline{MS}}(\mu_R^2) = \frac{12\pi}{(33 - 2n_f) \ln(\mu_R^2/\Lambda^2)} \left[1 - \frac{6(153 - 19n_f)}{(33 - 2n_f)^2} \frac{\ln \ln(\mu_R^2/\Lambda^2)}{\ln(\mu_R^2/\Lambda^2)} \right] \quad (16)$$

in our NLO predictions. The $\Lambda_{\overline{MS}}$ value is chosen consistent to the $\Lambda_{\overline{MS}}^{(4)}$ value from the parton distribution functions. The value of α_s is matched at the thresholds $q = m_q$ and the number of flavours n_f in α_s is fixed by the number of flavours for which the masses are less than μ_R . Furthermore the number of quark flavours that can be pair-produced are set equal to n_f chosen in α_s . In all predictions, the renormalization scale and the factorization scale are set to be equal: $\mu_R^2 = \mu_F^2 = \mu^2$. For the fine structure constant α we adopt the running coupling formula.

The following kinematical cuts are used for the HERA results:

$$\begin{aligned} 0.001 &< x < 0.1 \\ 0.04 &< y < 0.95 \\ 600 \text{ GeV}^2 &< W^2 \end{aligned} \quad (17)$$

In addition, we use two different Q^2 ranges:

"Low Q^2 range:" $4 \text{ GeV}^2 < Q^2 < 100 \text{ GeV}^2$

"high Q^2 range:" $100 \text{ GeV}^2 < Q^2$

Note that the kinematical cuts in eq. (17) are not independent, for example the x and y cut also imply $W > 55 \text{ GeV}$ ($W^2 = (1 - x)ys$) and x_{max} in the low Q^2 range is $x_{max} = 0.0287$ rather than 0.1 ($Q^2 = xys$).

Our kinematical range for the E665 experiment is defined by [3]:

$$\begin{aligned} 0.003 &< x \\ 0.08 &< y < 0.95 \\ 4 \text{ GeV}^2 &< Q^2 < 25 \text{ GeV}^2 \\ 400 \text{ GeV}^2 &< W^2 \end{aligned} \quad (18)$$

In fig. 1 we show the dependence of the (2+1) jet rate on the resolution parameter y_{cut} for the two different Q^2 ranges at HERA (a,b) and for the E665 experiment (c). Solid (dashed) lines correspond to NLO (LO) predictions. The renormalization scale μ_R^2 and the factorization scales μ_F^2 are set equal to Q^2 (lower curves) and p_T^2 (upper curves). The transverse momentum p_T of the jets is defined in eq. (19). One observes, that the NLO corrections lower the (2+1) jet rate by about 15%. At $y_{cut} = 0.02$ the (2+1) jet rate exceeds about 4% (12%) for the low (high) Q^2 range at HERA. For the E665 experiment, the (2+1) jet rate exceeds about 10-11% for $y_{cut} = 0.04$. In the following, we use $y_{cut} = 0.02$ for HERA and $y_{cut} = 0.04$ for E665 as our standard values.

To get a feeling for the theoretical uncertainties originating from the choice of the renormalization and factorization scales, we show numerical results for the (2+1) jet cross sections for different μ^2 values in table 1. Let us first comment on the choice of the scales. In DIS scattering it is natural to take $\mu^2 = \mu_R^2 = \mu_F^2 = Q^2$. However, for jet production, the transverse momentum p_T of the jets should also be considered as a relevant scale. The transverse momentum is defined with respect to the γ^* direction. For the (1+1) case, one has two jets, the remnant jet and the struck parton jet, both with zero p_T [§]. In our case of (2+1) jet production, one expects two partonic jets with nearly opposite large p_T and the remnant jet at $p_T = 0$. For the LO processes, p_T is given by:

$$p_T^2 = Q^2 \frac{1-x_p}{x_p} z(1-z) \quad (19)$$

with

$$x_p = \frac{Q^2}{2pq} = \frac{x}{\eta} \quad z = \frac{pp_1}{pq} \quad (20)$$

where p_1 is the four momentum of one of the final partons. Finally, in analogy to the jet analysis in e^+e^- experiments one may use $y_{cut}W^2$ (see. eq. (14)) as a possible scale. To avoid to small scales for perturbation theory, the scales are clipped at a minimum value of 2 GeV². Varying $\mu^2 = \mu_R^2 = \mu_F^2$ between Q^2 , $0.25 p_T^2 - 4 p_T^2$ and $y_{cut} W^2$ induces an uncertainty of roughly $\pm 17\%$ ($\pm 4\%$) in LO (NLO) for the high Q^2 range at HERA and $\pm 20\%$ ($\pm 10\%$) in LO (NLO) for the E665 experiment. Therefore, the uncertainty in the theoretical predictions is markedly reduced by the NLO corrections.

In figs. 2-10 we show the dependences of the total cross section and the (2+1) jet cross sections as well as the (2+1) jet rate on the basic kinematical variables x, W^2 and Q^2 . Figs. a (b) are for the low (high) Q^2 range at HERA and c shows

[§] The intrinsic transverse momentum of the partons in the target is neglected

predictions for the E665 experiment. LO results are given in figs. 2,3,6,7,11-15 whereas figs. 4,5,8,9 and 10 show NLO predictions.

In order to estimate the theoretical uncertainty from the choice of the scales, all results are given for $\mu^2 = \mu_R^2 = \mu_F^2 = a p_T^2$ ($a=1/4, 1, 4$: solid lines), $\mu^2 = a Q^2$ ($a=1/10, 1, 10$: dashed lines) and $\mu^2 = y_{cut} W^2$ (dotted lines).

Let us first comment on the W distributions shown in figs. 2-5. The (2+1) jet rate decreases with increasing W . This is mainly a reflection of our jet definition in eq. (14) where the required invariant mass of the jets increases with increasing W^2 . The (2+1) jet rate for the high Q^2 sample is larger than for the low Q^2 range since the larger Q^2 causes at the average larger transverse momenta of the participating partons (see eq. (19)) and larger invariant masses. Comparing figs. 2,3 and figs. 4,5 one observes again that the uncertainty from the choice of the scales in the LO predictions is very large (figs. 2,3) whereas this uncertainty is markedly reduced by including the NLO corrections (figs 4,5).

Turning now to the x distributions one observes a quite different behaviour of the cross sections for the different kinematical ranges in figs. 6 and 8. The x distributions are mainly governed by the behaviour of the respective parton distributions in the allowed kinematical regions. Note that $x_{max} = 0.0287$ for the low Q^2 range (figs 6a and 8a). The (2+1) jet rates in figs. 7 and 9 are increasing with increasing x . This is mainly an effect of the increasing (2+1) jet phase space in our jet definition scheme (for a detailed discussion of the $O(\alpha_s)$ (2+1) jet phase space see [5]). Comparing the distributions in figs. 6-9, one observes again that the NLO predictions in figs. 8,9 are much less scale dependent than the LO predictions in figs. 6,7.

Fig. 10 shows NLO predictions for the (2+1) jet rate as a function of $\sqrt{Q^2}$. Results are given for $\alpha_s(\mu_R^2 = Q^2)$ (lower solid curve) $\alpha_s(\mu_R^2 = p_T^2)$, (upper solid curve) and $\alpha_s = \text{fix} = 0.25$ (dotted curve). Note that the (2+1) jet rate is an increasing function with increasing Q^2 . Therefore the Q^2 or p_T^2 dependence in α_s is overcompensated by the increasing (2+1) jet phase space relative to the total cross section in our jet definition scheme. One observes sizable differences between the predictions using a constant α_s and scale dependent (two-loop) α_s . Therefore a clear discrimination between the solid and dotted curves should be possible with the expected event statistics at HERA.

Note also, that $\mu^2 = p_T^2$ tends to predict larger (2+1) jet rates and differential distributions than $\mu^2 = Q^2$ (see also table 1 and figs. 1-10). This is in particular true for lower W^2 and higher x values.

In order to explore the p_T dependence of the jet rates in more detail, fig. 11 shows results for the (2+1) jet rate as function of an additional p_T cut. Applying a p_T cut of 4 GeV reduces the (2+1) jet rate for the low (high) Q^2 range at HERA from $\sim 4\%$ to 2-3% ($\sim 12\%$ to $\sim 10\%$) whereas the (2+1) jet rate falls

from 10% to less than 1% for the FERMILAB experiment. Fig. 11c shows also, that more than 50% of the jets at $\sqrt{s} = 30$ GeV are produced with a transversal momentum less than 2 GeV. However, these values are too small to allow for reliable predictions in fixed order perturbation theory and resummation techniques should be used in this kinematical region. In fact, the experimental results presented by the E665 collaboration [3] are significantly higher than our second order predictions in figs. 1-9 c.

In table 2, we give results for jet cross sections with an additional $p_{T\min}$ cut. Figs. 12-15 show the corresponding W and x distributions. Note that the (2+1) jet rates in figs. 12-15 are less x and W dependent than our results without the additional $p_{T\min}$ cut. This demonstrates that the increase of the jet rates for small W (figs. 3,5) and large x values (figs. 7,9) is mainly an effect of low p_T jets. This is also clear by an inspection of eq. (19).

V. SUMMARY

The (2+1) jet production in DIS is calculated up to NLO in perturbative QCD using an invariant jet definition scheme. The theoretical uncertainties originating from variations of the renormalization/factorization scales are well under control. Differential distributions of (2+1) jet rates in W^2 , x and Q^2 are presented. A comparison with the large number of events expected at HERA in the near future will allow for precision tests of perturbative QCD. It is shown that an additional p_T cut to our jet definition scheme is necessary to obtain reliable predictions for jet production at the energy of the E665 experiment at FERMILAB.

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TABLES

μ^2	low Q^2 : LO	low Q^2 : NLO	high Q^2 : LO	high Q^2 : NLO	E665: LO	E665: NLO
Q^2	2360	1930	600	547	1200	1020
$10 \cdot Q^2$	1980	1930	472	502	879	932
$0.1 \cdot Q^2$	3050	2000	770	548	1660	1130
$y_{cut}W^2$	2080	1960	602	553	1040	983
p_T^2	2320	2020	719	579	1310	1120
$4 \cdot p_T^2$	2100	2000	615	562	1040	1010
$0.25 \cdot p_T^2$	2670	2140	857	583	1590	1180

TABLE I. (2+1) jet cross sections in [pb] for different choices of the renormalization and factorization scales (column 1). LO and NLO results are shown for the “low Q^2 ” (column 2 and 3) and the “high Q^2 ” (column 4 and 5) ranges at HERA ($y_{cut} = 0.02$) and for the E665 experiment (column 6 and 7) ($y_{cut} = 0.04$).

μ^2	low Q^2	high Q^2	E665
p_T^2	1140	451	75
$4 \cdot p_T^2$	1040	394	59
$0.25 \cdot p_T^2$	1230	514	103

TABLE II. LO predictions for (2+1) jet cross sections in [pb] with an additional p_{Tmin} cut of 4 GeV. The two-loop formula for α_s is used in the calculation.

FIGURE CAPTIONS

- Fig. 1. (2+1) jet fraction versus the cut variable y_{cut} for the low and high Q^2 range at HERA (a,b) and for the E665 experiment (c). The kinematical ranges are defined in eqs. (17,18). Solid (dashed) lines correspond to NLO (LO) predictions. In the upper (lower) lines the renormalization scale μ_R and factorization scale μ_F are set equal to $\mu^2 = p_T^2$ ($\mu^2 = Q^2$).
- Fig. 2. W dependence of the total cross section (upper line) and the (2+1) jet cross section in LO for the low Q^2 (a) and high Q^2 (b) range at HERA and for the E665 experiment (c). $y_{cut} = 0.02$ (0.04) in a,b (c). The different curves for the jet cross section belong to different choices of the renormalization scale μ_R and factorization scale μ_F :
solid lines: $\mu^2 = \mu_R^2 = \mu_F^2 = 1/4p_T^2, p_T^2, 4p_T^2$ (from top to bottom).
dashed lines: $\mu^2 = 0.1Q^2, Q^2, 10Q^2$ (from top to bottom).
dotted line: $\mu^2 = y_{cut}W^2$.
- Fig. 3. W dependence of the (2+1) jet rate in LO. Solid, dashed and dotted lines as in fig. 2.
- Fig. 4. same as fig. 2, but NLO predictions. The two upper lines in c) show the LO (dashed) and NLO ($O(\alpha_s)$, solid) predictions for the total cross section. The difference between these results for HERA energies are too small to be visible in a) and b).
- Fig. 5. same as fig. 3, but NLO predictions.
- Fig. 6. x dependence of the total cross section (upper line) and the (2+1) jet cross section in LO for the low Q^2 (a) and high Q^2 (b) range at HERA and for the E665 experiment (c). Solid, dashed and dotted curves as in fig. 2.
- Fig. 7. x dependence of the (2+1) jet rate in LO. Solid, dashed and dotted curves as in fig. 6.
- Fig. 8. same as fig. 6, but NLO predictions.
- Fig. 9. same as fig. 7, but NLO predictions.
- Fig. 10. $\sqrt{Q^2}$ dependence of the (2+1) jet rate in NLO for $\mu^2 = Q^2$ (lower solid line) $\mu^2 = p_T^2$ (upper solid line). Also shown is the result for $\alpha_s = \text{const.} = 0.25$ (dotted line).
- Fig. 11. LO (2+1) jet rate as a function of p_{Tmin} for the low and high Q^2 range at HERA with $y_{cut} = 0.02$ (a,b) and for E665 experiment (c) ($y_{cut} = 0.04$).

The three curves are for $\mu^2 = \mu_R^2 = \mu_F^2 = 1/4p_T^2, p_T^2, 4p_T^2$ (from top to bottom). We have used the two-loop formula for α_s .

Fig. 12 W dependence of the total cross section (upper line) and the (2+1) jet cross section in LO for the low Q^2 (a) and high Q^2 (b) range at HERA and for the E665 experiment (c) with an additional p_{Tmin} cut of 4 GeV (for the jet cross sections). Same parameters as in fig. 11 for the (2+1) jet cross sections.

Fig. 13 W dependence of the (2+1) jet rate in LO with an p_{Tmin} cut of 4 GeV (parameters as in fig. 11).

Fig. 14. x dependence of the total cross section (upper line) and the (2+1) jet cross section in LO for the low Q^2 (a) and high Q^2 (b) range at HERA and for the E665 experiment(c) (parameters as in fig. 11).

Fig. 15. x dependence of the (2+1) jet rate in LO with an p_{Tmin} cut of 4 GeV (Parameters as in fig. 11).

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